Aufgabe 20

$$g = \left[\begin{pmatrix}1\\1\\1\end{pmatrix}\right] =: U_1 \qquad h = \begin{pmatrix}0\\0\\2\end{pmatrix} + \left[\begin{pmatrix}1\\0\\0\end{pmatrix}\right] =: U_2$$

ges.: $n \perp (U_1 + U_2)$ mit $\|n\| = 1$

$$\Rightarrow \quad n := \frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\-1\end{pmatrix}$$

$$\operatorname{dist}(g,h) = |n \cdot (0,0,2)^T| = \left|-\frac{2}{\sqrt{2}}\right| = \sqrt{2}$$

Aufgabe 21

$$\mathcal{H} := H\left(\begin{pmatrix}2\\1\\1\\1\end{pmatrix}, \begin{pmatrix}2\\i\\0\\0\end{pmatrix}, \begin{pmatrix}1\\0\\1\\i\end{pmatrix}, \begin{pmatrix}1\\-i\\1\\0\end{pmatrix}\right)$$

$$= \begin{pmatrix}1\\0\\1\\i\end{pmatrix} + \underbrace{\left[\begin{pmatrix}1\\1\\0\\1-i\end{pmatrix}, \begin{pmatrix}1\\i\\-1\\-i\end{pmatrix}, \begin{pmatrix}0\\-i\\0\\-i\end{pmatrix}\right]}_{=: U}$$

$$U = \begin{pmatrix}1 & 0 & 1\\ 0 & 1 & 1\\ -1 & 0 & 0\\ -2i & 1 & 1-i\end{pmatrix} \mapsto \begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 1\\ -1 & 0 & 1\\ -2i & 1 & 1+i\end{pmatrix}$$

$$\mapsto \begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ -1 & 0 & 1\\ -2i & 1 & i\end{pmatrix}$$

$$\mapsto \begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\\ -i & 1 & i\end{pmatrix}$$

$\Rightarrow U^0 = [(i, 1, -i, 1)] =: [n^*]$

$\langle n^*, x-s\rangle \, \mathrm{Re} = 0$

$\Leftrightarrow \quad n^* \cdot (x-s)\, \mathrm{Re} = 0$

$\Leftrightarrow \quad (\overline{n^*})^T \cdot (x-s)\, \mathrm{Re} = 0$

$\Rightarrow [n] = \begin{pmatrix}-i\\1\\i\\1\end{pmatrix} \quad \Rightarrow \quad n = \frac{1}{2}\begin{pmatrix}-i\\1\\i\\1\end{pmatrix}$

$\Rightarrow \mathcal{H}: \frac{1}{2}\begin{pmatrix}-i\\1\\i\\1\end{pmatrix} \cdot \begin{pmatrix}x_1-2\\x_2-1\\x_3-1\\x_4-1\end{pmatrix} = 0$

$\mathrm{dist}(\mathcal{H}, p) = |n \cdot (p - s)|$

$= \left| \frac{1}{2}\begin{pmatrix}-i\\1\\i\\1\end{pmatrix} \cdot \begin{pmatrix}i-2\\i-1\\0\\0\end{pmatrix} \right|$

$= \left| \frac{1}{2}(-1-2i+i-1) \right|$

$= \left| -1 - \frac{i}{2} \right| = \frac{3}{2}$

Aufgabe 22

$$\pi : \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \mapsto 7x_1^2 + 2x_1x_2 - 14x_1 + 7x_2^2 - 2x_2 - 9$$

a) $$q : \begin{pmatrix} x_0 \\ x_1 \\ x_2 \end{pmatrix} \mapsto 7x_1^2 + 2x_1x_2 - 14x_0x_1 + 7x_2^2 - 2x_0x_2 - 9x_0^2$$

$$\sigma\left(\begin{pmatrix} x_0 \\ x_1 \\ x_2 \end{pmatrix}, \begin{pmatrix} y_0 \\ y_1 \\ y_2 \end{pmatrix}\right) = 7x_1y_1 + \cancel{2}x_1y_2 + x_2y_1 - 7x_0y_1 - 7\cancel{y}x_1y_0 + 7x_2y_2 - x_0y_2 - x_2y_0 - 9x_0y_0$$

b) $$g := \left(\begin{array}{c|cc} -9 & -7 & -1 \\ \hline \boxed{\begin{matrix} -7 \\ -1 \end{matrix}} & \multicolumn{2}{c}{\underbrace{\begin{matrix} 7 & 1 \\ 1 & 7 \end{matrix}}_{=:G}} \end{array}\right)$$

$$(m_1, m_2) \cdot G = (7, 1) \quad \Leftrightarrow \quad G^T \cdot \begin{pmatrix} m_1 \\ m_2 \end{pmatrix} = \begin{pmatrix} 7 \\ 1 \end{pmatrix}$$

$$\cancel{\Leftrightarrow (m_1, m_2) = (7,1) \cdot G^{-1} = (7,1) \cdot \frac{1}{48} \begin{pmatrix} 7 & -1 \\ -1 & 7 \end{pmatrix} =}$$

$$\Leftrightarrow \quad \begin{array}{cc|c} m_1 & m_2 & 1 \\ \hline 7 & 1 & 7 \\ 1 & 7 & 1 \\ \hline 0 & -48 & 0 \\ 1 & 7 & 1 \\ \hline 1 & 0 & 1 \\ 0 & 1 & 0 \end{array} \quad \begin{array}{l} \\ \cdot(-7) \\ \\ \cdot \frac{1}{-48} \\ \cdot(-7) \\ \\ \\ \end{array}$$

c) EW u. G:

$$\det(G - xE_2) = \begin{vmatrix} 7-x & 1 \\ 1 & 7-x \end{vmatrix} = (7-x)^2 - 1 = 48 - 14x + x^2$$
$$= (x-8)(x-6)$$

$$\Rightarrow \lambda_1 = 8, \quad \lambda_2 = 6$$

EV zu $\lambda_1 = 8$:

$$\left.\begin{matrix} -1 & 1 \\ 1 & -1 \end{matrix}\;\right|\; 0 \quad \Rightarrow \quad \tilde{b}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\Rightarrow \tilde{b}_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$